A Simulation of Segregation in Cities and its Application for the Analysis of Price Regulation

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Abstract

Social segregation in cities takes place where different household groups exist and when, according to Schelling, their location choice either minimizes the number of differing households in their neighborhood or maximizes their own group. In this contribution an evolutionary simulation based on a monocentric city model with externalities among households is used to discuss the spatial segregation patterns of four groups. The resulting complex spatial patterns can be shown as graphic animations. They can be applied as initial situation for the analysis of the effects a price regulation has on segregation.

JEL Classification: D62, R14, R31, R52.
Keywords: Simulation, segregation, monocentric city, price regulation

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1 Introduction

Empirical observations of today's city structures show increasing segregation by ethnic or lifestyle groups (Sassen [10]; Harth, Herlyn, Scheller [6]; Schneider, Spellerberg [12]; Wagner [15]). Such an ethnic or other non-economic segregation takes place where different household groups exist and if there are either negative externalities between households of different groups or positive externalities among households of the same group. Racism as well as the existence of social networks are particular examples of these phenomena. According to Shelling [11], such externalities lead to a dynamic process of segregation because when households choose their location, they either minimize the number of differing households in their neighborhood or maximize their own group. This evolutionary process is called *tipping-process*.

With advancing computing possibilities, the simulation of evolutionary complex systems becomes more and more important for urban modelling (Torrens and O'Sullivan [14]). Especially the use of extended cellular automata provides interesting results. Portugali et al. [9] and Portugali [8] use cellular automata models to simulate segregation processes. As driving force they stipulate that social networks may be the reason for positive externalities among households with similar demographic or ethnic characteristics. As in Shelling's [11] *tipping-process*, their system is leading to patterns with strong segregation. However, their contributions are limited in two aspects. The location choice of households is explained in a behavioristic fashion only and the city possesses no other characteristics other than the initial distribution of the population. Thus, the size of the city is usually limited by the modelled space only. While the possibility to model externalities between households by using evolutionary approaches is shown, it is necessary to explain the size of the city in order to show the city structure endogenously.

In this contribution an evolutionary simulation based on a monocentric model is used to discuss the spatial segregation patterns of four household groups. The location choice of households is dependent on the distance to the nearest of one or two city centers. Furthermore, externalities between different types of households are being assumed which influence their preferences for a certain neighborhood. As a consequence the evaluation of a neighborhood depends on the respective type of household. In this model, the result of the allocation process is open. It is unclear whether or not the process leads to an equilibrium. The arising complex spatial patterns can be applied as initial situation for the analysis of the effects a price regulation has on segregation.

This paper is organized as follows: Section 2 presents a model containing a local public good, a constant elasticity of substitution (CES) production function responsible for households' income as well as externalities between households of different types. The model is solved within a simulation, closing with the allocation process used in the simulation. The section finishes with a simple run of the simulation showing the spatial patterns obtained. In Section 3, an application of the simulation is presented: the implementation of a price regulation which is abolished later on. Section 4 summerizes.

2 The Model

The model is basically an extended cellular automaton as in Batty [3], using an even surface, divided into l cells. The population is free to move between cells inside the city but also from outside into the city and vice versa. The allocation is simulated as an iterative process. In each iteration a certain share of the population is leaving a cell. The vacant land will be allocated to households with the highest bids. The process may lead to an equilibrium but the result is open. It is dependent on the expectations of households regarding income and externalities (figure 1). As a result, we obtain the distribution of population on the surface, as in common cellular automata models visualized in a plot which looks quite similar to a population map.

To simulate segregation, population is divided into different groups of households. These groups can be distinguished by family structure, race, lifestyle or other non-economic characteristics. The microeconomic foundation of the simulation is a monocentric model introduced by Alonso [1, 2], here presented in the *Muth* style [7, p. 37]. Accordingly households' behavior follows a utility function including housing, consumption and commuting to the city center. Apart from these goods, we are considering the influence of externalities between different households.

Since we are using an *open city* approach, the values of living inside or outside the city have to be compared. The key question here concerns a household's evaluation of an inner-city location compared to one outside the city with given characteristics at a given price. As in other monocentric models the answer lies in a bid-price function with a form which depends on the goods considered in the utility function, usually housing and a centrally offered consumption good. In this approach however, while the consumption good is obtainable everywhere, we regard two other arguments of utility, namely a centrally provided public good and externalities between households. Inside the city households are connected to the city center by consumption of the public good, while outside the city, spatial structure is assumed to be unimportant – think of more or less independent settlers. Apart from distance to the city center the value of a location depends on the

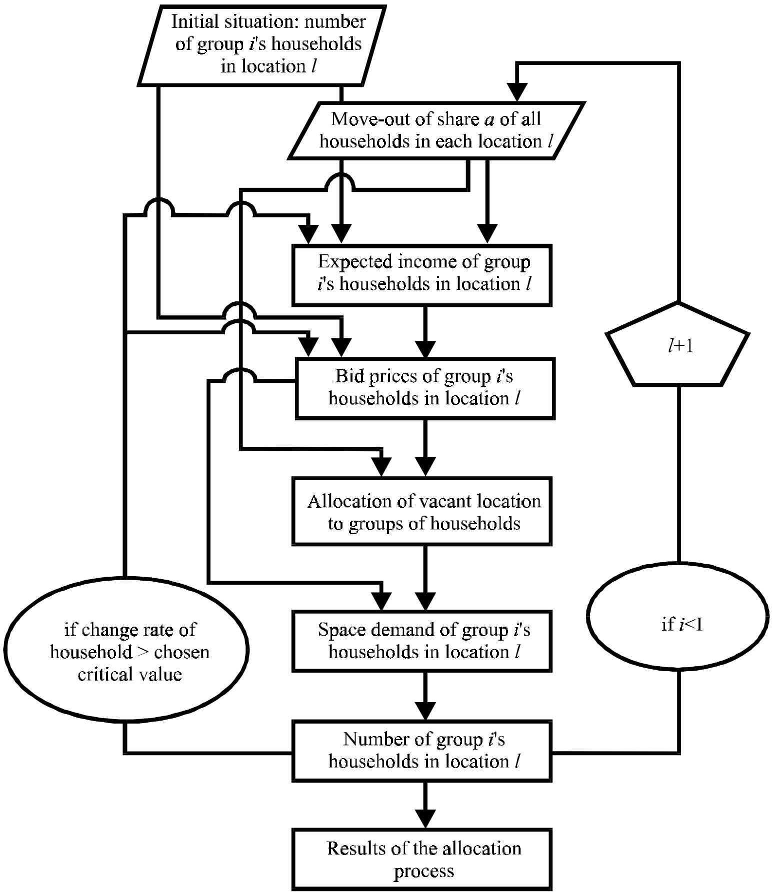

Figure 1: Allocation process

spatial distribution of other households.

By using a utility function of the Cobb-Douglas (CD) form, we obtain as bid-price function:

$$\psi_{i,l} = p_b \left(\frac{T}{tr_l}\right)^{\frac{\alpha_{z,i}}{\alpha_{s,i}}} \left(\frac{\widehat{Y}_i}{\overline{Y}_i}\right)^{\frac{\left(\alpha_{s,i}+\alpha_{x,i}+\alpha_{z,i}\right)}{\alpha_{s,i}}} \prod_{j=1} \left(\frac{n_{i,j,l}}{\overline{n}_{i,j,l}}\right)^{\frac{\alpha_{n,i,j}}{\alpha_{s,i}}} . \tag{1}$$

Here, commuting inside a city is necessary to receive a unit of a public good z for free. tr_l are commuting costs of a location with a distance r to the city center. Outside the city the public good is provided by the state and financed by a tax T per unit of public good. This tax may be included in income tax, a property tax, or be capitalized as a development fee. $\widehat{Y}_i$ is the expected income of a member of group i inside the city, and $\overline{Y}_i$ is obtainable outside. The externalities caused by members of group j are $n_{i,j,l}$ inside and $\overline{n}_{i,j,l}$ outside the city. The exponents $\alpha_{z,i}$, $\alpha_{s,i}$, $\alpha_{x,i}$ and $\alpha_{n,i,j}$ are exogenous and represent preferences of a member of group i for the goods in the index.

Thus, we realize that the willingness to pay depends on the advantage of a location inside compared to one outside the city. The additional tax for public goods outside the city is used in order to ease the model solution for a CD utility function.

To specify the bid-price function, we have to calculate externalities and income. The system becomes dynamic if we assume income and externalities as dependent on the population of the city. The first is plausible by definition of externalities. We are using a potential approach, standardizing externalities n to 1 unless there is an influence of the potential:

$$n_{i,j,l} = 1 + \sum_l \left(\frac{H_{j,l}}{\exp(d_{l,l'})}\right) \tag{2}$$

with $d_{l,l'}$ being the distance between locations l and l'. The case of $\overline{n} = 1$ is used as reference outside the city.

The foundations for the second relationship have to be explained. There is a broad discussion on agglomeration effects generated within urban production. In the present approach, these are caused by combining members of different household groups, leading to higher productivity and income. This we represent by using a CES production function, with H_i being labour offered by members of different groups and q_i being the quality of labour of a group:

$$Y_i = p_x q_i^\rho \left(\left(\sum_i \left(q_i H_{i,\varepsilon}\right)^\rho\right)^{1/\rho} \left(q_i H_{i,\varepsilon}\right)^{-1}\right)^{(1-\rho)} . \tag{3}$$

To obtain an income outside the city, it is assumed that only members of the same group, perhaps only a single household, are involved in production. In this case, the income is p_x.

Since income is dependent on population, households have to build expectations regarding their income in a certain period of time. The easiest way here is to use the concept of static expectations since tests showed that adaptive expectations would not improve the process and leave the results nearly unchanged. Unfortunately, using the CES production function leads to two specific problems. Firstly, if a groups' share of the population is small, there is an extreme increase in income, and secondly, if a group disappears, the expected income becomes zero. This edge solution can be avoided by assuming that there has to be a minimal share of one groups' population to gain information about their income. If the share of this group is larger than, say, ι, the expected income $\widehat{Y}_{i,\varepsilon}$ in period ε equals the income of the last period $\varepsilon - 1$. Otherwise it will be the average income of the whole cities' population of the last period. Income follows as:

$$\widehat{Y}_{i,\varepsilon} = \left| \begin{array}{c} \widehat{Y}_{i,\varepsilon-1} \text{ if } \frac{H_{i,\varepsilon-1}}{\sum_i H_{i,\varepsilon-1}} \geq \iota \\ \frac{\sum_i H_{i,\varepsilon-1} \widehat{Y}_{i,\varepsilon-1}}{\sum_i H_{i,\varepsilon-1}} \text{ if } \frac{H_{i,\varepsilon-1}}{\sum_i H_{i,\varepsilon-1}} < \iota \end{array} \right. . \tag{4}$$

Knowing the bid-price function and its elements, we can think about the allocation process itself. In the simulation the evolution of a city starts with the rise of a city center within a concentration of households, maybe a village, with randomly distributed households of different groups. For an allocation process it is necessary that a certain amount of locations is set free to be obtainable for new households. Assuming that households which moved in in one period move out in equal fractions within a certain time, say 10 periods, then in each period 10% of the land becomes vacant. This is easy to simulate and close enough to reality.

Landlords give vacant land to households with the highest bid for a dwelling if the housing market provides perfect allocation and if pricing is unregulated. To represent imperfections of allocation due to a search process or regulation, a parameter representing a *pricing accuracy* is introduced. It is the share ν of the highest bid which the landlords are able or allowed to realize. This parameter will have a value between 0 for total failure of pricing and 1 for perfect pricing. The bids follow the bid-price function (equation 1). Thus we obtain the allocation of land to household groups. Knowing the realized price, the share of land per household may be calculated for each location, and we obtain the allocation of households to locations $H_{i,l}$. This result of the housing market is the starting point for the next iteration.

Again people move out and release a share of land which can be reallocated to households.

There is no natural end to this iteration algorithm; it must be stopped by rule. A stop rule may either be a criterion of an equilibrium or a critical number of iterations. The relative change in the number of households may be used as a criterion of equilibrium. If the change is smaller than a chosen critical rate ι in each cell, the city seems to be in equilibrium.

Within the allocation process several results are obtainable: Household numbers per cell, income of different households, production of goods, profits of firms and landlords and ultimately welfare as the total landlords' surplus. The spatial patterns of segregation can be demonstrated by the graphical representation of a populations' distribution. Marking the cells of the simulated city in the color of the strongest group, we obtain a city map of population distribution. Comparing this map to the spatial distribution of each single group, we can see whether groups are concentrated or mixed with others. The density of household distribution can be shown by a three dimensional bar plot. For different parameter specifications we should expect different results.

The simulation was run on *matlab* for different parameter values and underwent an extensive sensitivity analysis (Wagner [16]). The initial parameter specification is oriented on empirical relations and on plausibility. In particular the parameters for the exponents of the utility function are based on data of household expenditures. According to the Cobb-Douglas utility function, the ratio of one exponent to the total of exponents shows the share of expenditures for this good or bundle of goods. The commuting costs are set according to practical reason. Within the observed cell space the commuting costs reach the cost of obtaining public goods outside the city. At this distance to the city center we expect the city border. Other parameters can be established by trial and error which is legitimate as long as we only aim at understanding the work of the system[1].

The first calibration presented in this contribution is applied to four groups of households with similar utility functions and one city center. In figure 2 it is obvious that this first setting leads to a complex spatial pattern which does not look ring-shaped as we would expect for static monocentric city models. However, figure (3) shows a falling density gradient of household distribution, as in such models (Muth [7, p. 71]).

If we take a look at the dynamic allocation process, which also could be

[1]The initial parameter values are as follows: 20x20 Cells, $d_{l,l'} = 1$ for l and l' being direct neighbors, $\alpha_{x,i} = 0.3$, $\alpha_{s,i} = 0.15$, $\alpha_{z,i} = 0.1$, $\alpha_{n,i,i} = 0.0075$, $\alpha_{n,i,j} = 0$, $\rho = 0.99$, $q_i = 1$, $p_x = 0.1$, $p_b = 10$, $T = 5$, $t = 1$, $\iota = 0.001$ and $\nu = 1$.

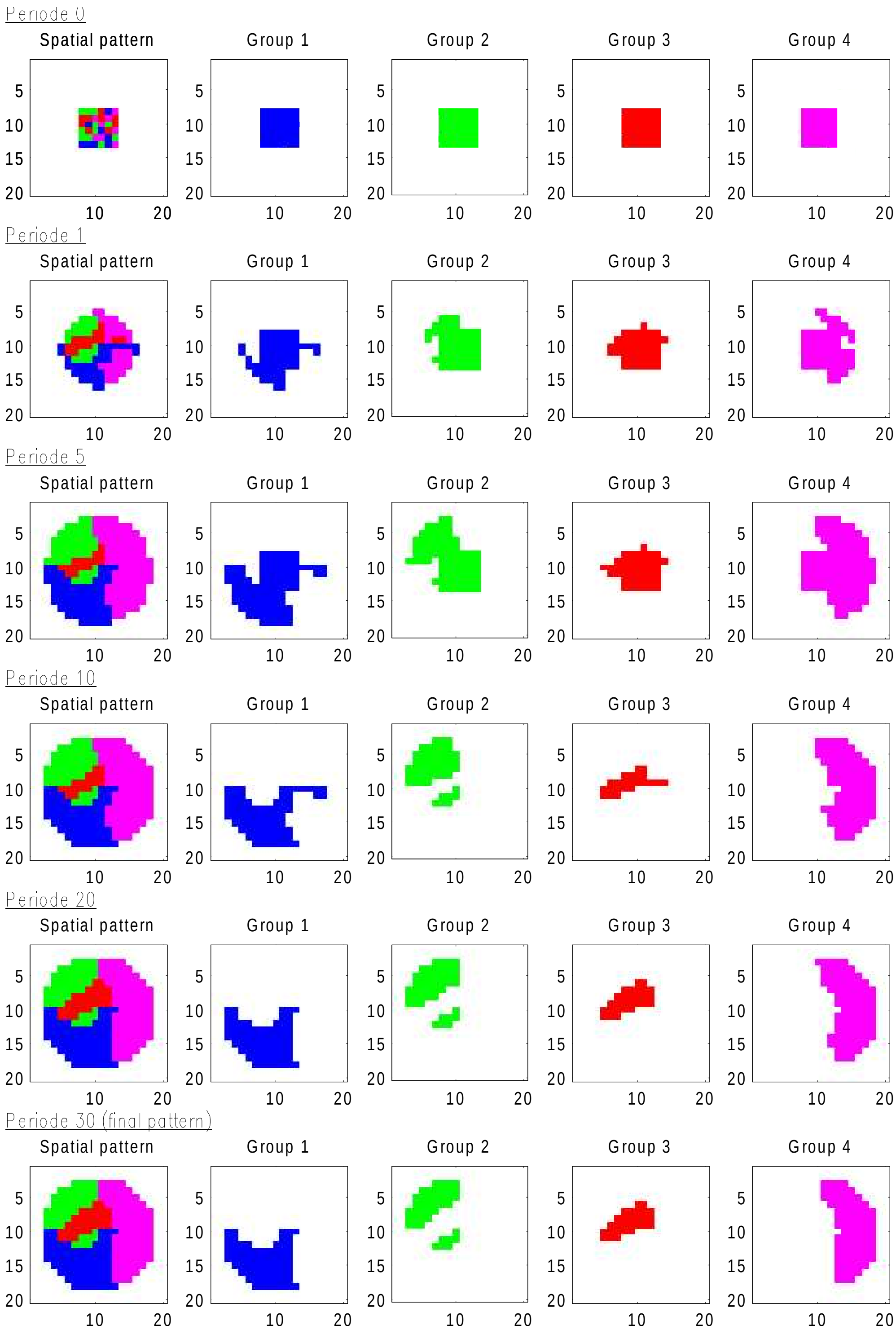

Figure 2: Development of spatial distribution of four household groups with symmetric socioeconomic variables in a monocentric city

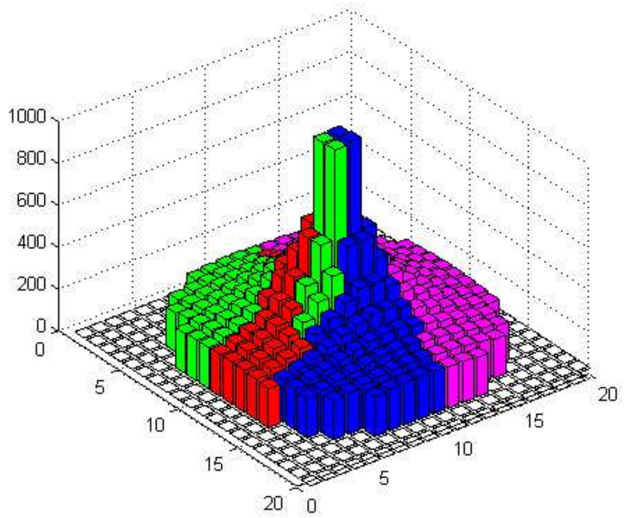

Figure 3: Density of four household groups with symmetric socioeconomic variables in a monocentric city

presented animated on computer screen, we can follow the development of a city from a square-shaped village with randomly distributed population (figure 2). Usually we observe an instantaneous growth following the establishment of a city center. The final size of the city is quickly reached while the segregation process follows growth. For different initial situations an equilibrium is usually reached within 50 periods, whereas the spatial growth needs only a few periods. Depending on the rate of moves, an iteration may be counted as a one or two years' period. Thus the equilibrium is to be understood only as a direction of the allocation process which usually will be affected by exogenous shocks before equilibrium is reached.

Changing variables, we realize that these results are quite sensitive. In open cities a mixed city only emerges if income is positively related to the population mix. Otherwise, only one group will settle within the city while the others are being expelled. Due to the static expectations regarding income and neighborhood for some parameter constellation, we obtain a fluctuating system without an equilibrium. The reason for this is obvious: If a group is very small and consequently its income is very high, this leads to high immigration into the city, thereby diminishing the sizes of other groups in every location of the city. Thus, formerly large groups become smaller ones with high income and the process turns round. An equilibrium is possible only if none of the groups becomes small enough for its members to expect extremely high income. This leads to segregated patterns.

Varying the parameters within a certain range leading to a segregated pattern, we usually obtain results which reach our expectations: As long as the parameters are symmetric for both groups, asymmetries of the spatial structure are caused only by differences in the initial distribution of population. A productivity rise as well as of positive externalities increases the city's population and size. Both aspects aim at the advantage of living in the city rather than at an outside location. On the other hand, if we raise commuting costs or the preferences for consumption goods, the size of the city decreases. If for instance there were clusters, these might cause higher externalities and lead to larger areas in the equilibrium. However, we may vary the preferences for externalities, goods or housing. In such cases, a groups' population and area are larger the higher it values locations within the city compared to locations outside.

3 Application: Price Regulation

Patterns reached through simulations may be used as initial situations for simulating policies and their implications for the spatial structure of a city. The assumptions for the setting presented in the previous section are quite rigorous compared to empirical findings. There, groups are assumed to be symmetric in socioeconomic variables although usually they will differ in their ability to establish social networks, in their education and so forth. Thus, for applications and policy analyses it seems appropriate to generate a more realistic but also a more arbitrary setting. Therefore a second setting is created as follows: Two groups (group 2 and 3) remain as before. For another group (group 1), now, a lower level of education, a weaker social network but higher preferences for centrally provided public goods are assumed. This assumption hints at a group with socioeconomic disadvantages compared to the reference groups 2 and 3. The fourth group instead is assumed to be highly educated and with lower preference for centrally provided public goods, indicating at a group with economic advantages. Their ability to establish social networks remains as before[2].

In addition to the changed socioeconomic structure of population a change in geography is assumed. Instead of one city center, two are assumed located a bit north-west and south-east of the former.

The spatial pattern following this setting is shown in figure 4. The group with higher education and weak preferences for centrally provided public

[2] The changed variables are $q_1 = 0.8$, $\alpha_{n_1} = 0.0025$, $\alpha_{z_1} = 0.12$, $q_4 = 1.2$ and $\alpha_{z_1} = 0.08$.

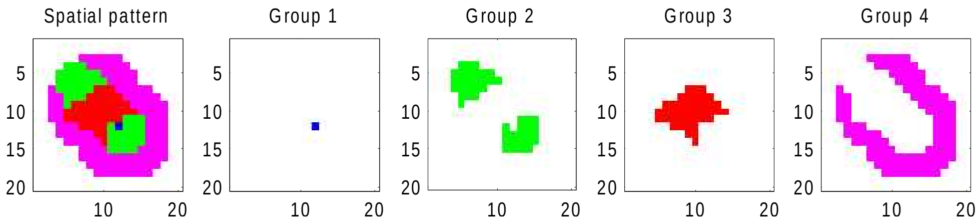

Figure 4: Spatial distribution of four household groups with different education and preferences for central public goods and neighborhood in a city with two centers

goods forms a nearly ring-shaped area around the other groups with higher preferences which are located closer to the city centers. The reason for this is that the bid-price function of the group with lower preferences is less steep than the others. Thus, ring-shaped patterns may be obtained due to variation of parameters which are responsible for the relationship to the city center. In this case the exponent of the centrally offered public good in the utility function is changed asymmetricly. Since the bid-price function of the group with lower preferences for centrally provided public goods is lower than the other, the well-known intersection of the two bid-price functions arises (Fujita [4, 5]), dividing the areas of the groups. In this case the effect of the city center dominates the effect of neighborhood clusters.

The group with lower education, a weak social network and high preferences for centrally provided public goods merely settles in a small area close to one city center. The two other groups surround the city centers. All in all, as in the first setting perfect segregation is observable.

Using this spatial structure as an initial situation, the implementation of a price regulation is simulated. As price regulation it shall be considered that the landlord is allowed to realize only a share of the highest bid for housing services at a location. Such a regulation is quite similar to the German price regulation for rented housing. In the simulation, this is implemented by varying the parameter of pricing accuracy ν.

If the pricing accuracy is forced to fall to 0.75 the spatial pattern changes radically. The group with competitive disadvantages may now be able to afford housing services in a larger area. The areas of the other groups also increase. The different groups now settle in the same area, thus reducing segregation. As an exception few areas at the city border remain where only highly educated households with low preferences for centrally provided public goods live.

As a first important result it is shown that price regulation is an opportunity to avoid or reduce segregation, the reason being that the mechanism of establishing segregation, the market mechanism, is disturbed.

The cost of such a policy is quite obvious: Welfare provided by this market, counted as landlords' surplus, is reduced because the surplus is the sum of realized prices for housing which are reduced by the regulation. This may be an argument for a policy of deregulation. The consequences of deregulation can be shown by using the spatial pattern gained by deregulation as initial situation for subsequent simulation with perfect pricing accuracy ($\nu = 1$). The resulting spatial structure is shown in figure 6. Here, we can see that perfect segregation is reestablished. The fourth group again occupies a ring-shaped area at the border of the city while the second and third group settle close to city centers. However, in contrast to the situation before regulation, the group of households with socioeconomic disadvantages disappears. They cannot meet the competition and have to leave the city.

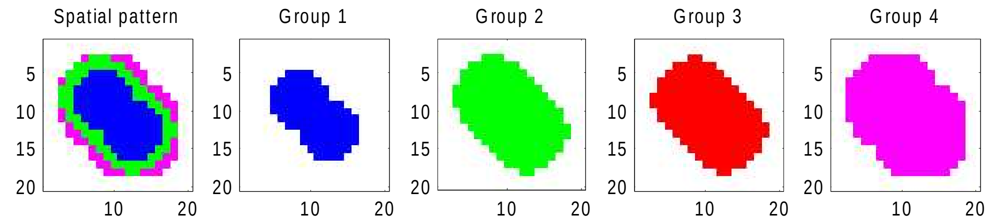

Figure 5: Spatial distribution of four household groups with differing education and preferences for central public goods and neighborhood in a city with two centers after implementation of price regulation ($\nu = 0.75$)

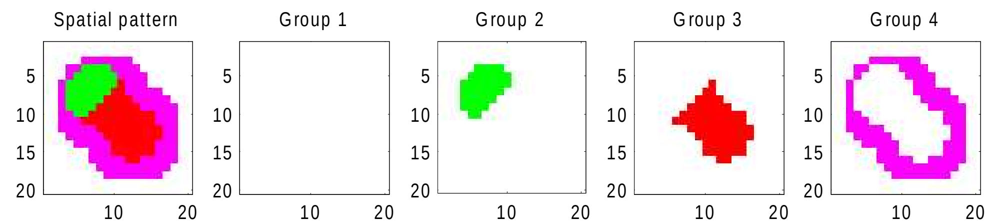

Figure 6: Spatial distribution of four household groups with differing education and preferences for central public goods and neighborhood in a city with two centers after deregulation ($\nu = 1$)

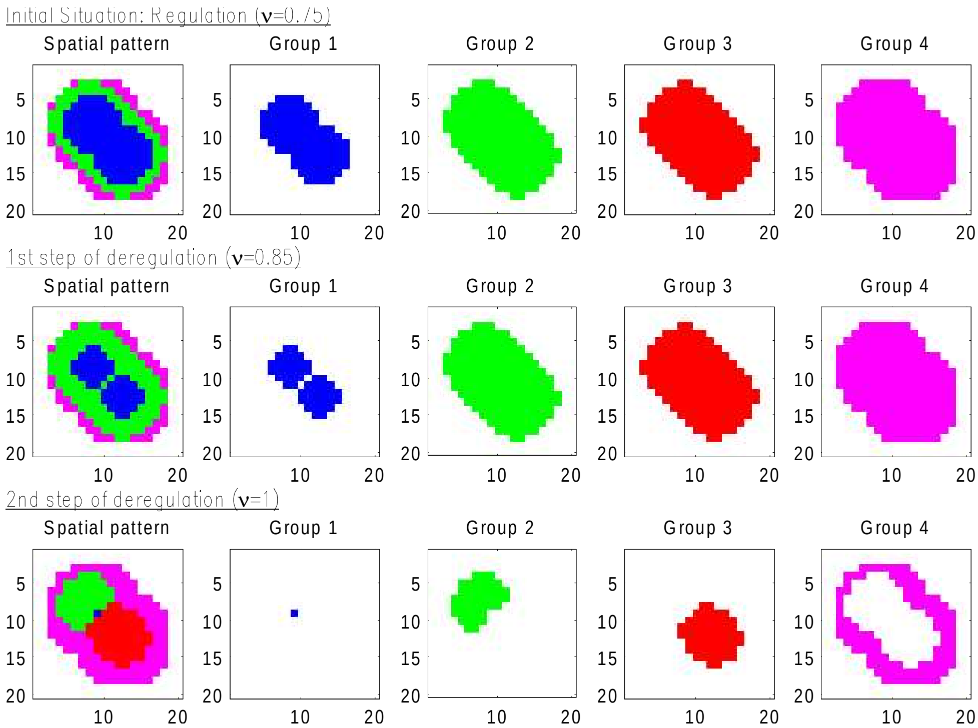

Figure 7: Spatial distribution of four household groups with differing education and preferences for central public goods and neighborhood in a city with two centers after gradual step by step deregulation (1st step: $\nu = 0.85$; 2nd step: $\nu = 1$)

Obviously the process is path dependent, and it is not clear that the patterns return to their original structures if parameters are changed to their initial values. This extreme result can be avoided if the deregulation is established more carefully. In figure 7 a gradual step by step deregulation is presented. As a first step, the pricing accuracy is allowed to rise to $\nu = 0.85$. A rise to $\nu = 1$ is allowed as a second step after the new spatial structure has stabilized. We can observe that in this case the group with socioeconomic disadvantages may preserve a small area close to a city center, as in the pattern before the regulation was implemented.

We can see the reason for the differing results in figure 8. In both cases we observe a rapid decrease of the disadvantaged group (group 1) caused by increased competition. As consequence the area of this group shrinks without reaching the concentration of the initial situation before any regulation. This process needs 10 periods due to the move-out scheme. In the meantime no vacant location is given to a household of the disadvantaged group. With

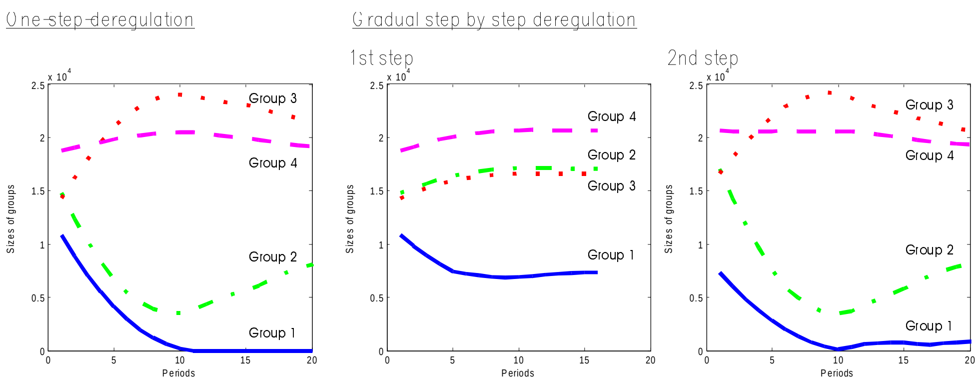

Figure 8: Development of household numbers after deregulation

gradual deregulation such a decrease can be noticed after each step. In this case, after 10 periodes the number of households increases again and stabilizes at a certain level. It is caused by the positive effect the decreased household number has on income. This enables the disadvantaged households to compete in the area still inhabited by a few members of their group, thereby generating positive externalities. In the case of deregulation in one step the group is expelled because even the increase in income does not compensate the rapidly rising prices. Due to their lower concentration the disadvantaged households do not generate sufficient positive externalities. This is the reason why they cannot compete although they had been able to do so in the initial situation with the same parameters as before any regulation. This result shows that policy changes may disturb established social networks, which stabilize the spatial patterns of a city.

4 Conclusion

In this contribution, with a simulation, complex patterns of spatial segregation caused by social networks of households are presented. The underlying model is a modified monocentric model with positive externalities among households. In a simple run of the simulation the spatial pattern shows perfect segregation. As long as household groups possess symmetric socioeconomic variables, the pattern cannot be ring-shaped due to bid-rent functions being of the same steepness towards city centers.

As an application the implementation of price regulation is simulated for a city with two centers and a heuristic set of assumptions for household groups. The result is reduced segregation and larger areas available for households with socioeconomic disadvantages. Falling landlords' surplus is a consequence.

When in this situation regulation is abolished, the segregation reestablishes. Additionally a group with socioeconomic disadvantages cannot meet the competition and has to leave the city – a consequence of the path-dependency of spatial patterns. If, instead of a total deregulation, a gradual deregulation is implemented, this extreme result may be avoided even if in the end prices are set totally deregulated.

The phenomenon of displacement of groups with socioeconomic disadvantages seems to be of high relevance for housing policy. Other studies of the housing market (e.g. Tucker [14]) show that regulation leads to a gap in supply which is a consequence of reduced landlords' surplus. In this contribution, deregulation leads to higher segregation and less participation of households with socioeconomic disadvantages in the housing market. Additionally, changes in policy can disturb established social networks which offer positive externalities and thereby stabilize the spatial patterns of a city. To solve this dilemma, a political evaluation of segregation is needed to optimize welfare by balancing market efficiency, landlords' surplus and segregation. Policy changes, especially price deregulation, should be implemented carefully to avoid disadvantaged households being excluded from the housing market.

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